Chiral Perturbation in the Hidden Local Symmetry and Vector Manifestation of Chiral Symmetry [*]

Masayasu Harada

Department of Physics, Nagoya University, Nagoya 464-8602, JAPAN

Abstract

In this talk I summarize our recent works on the chiral phase transition in the large flavor QCD studied by the hidden local symmetry (HLS). Bare parameters in the HLS are determined by matching the HLS with the underlying QCD at the matching scale through the Wilsonian matching. This leads to the vector manifestation of the Wigner realization of the chiral symmetry in which the symmetry is restored by the massless degenerate pion (and its flavor partners) and rho meson (and its flavor partners) as the chiral partner.

Chiral phase transition in QCD is discussed in various contexts such as the large flavor QCD and the hot and/or dense QCD, etc. Recently, in Refs. 1, 2 and 3, we studied the chiral phase transition in the large flavor QCD using the hidden local symmetry (HLS) model [4], which is an effective field theory of QCD including vector and pseudoscalar mesons.

The chiral symmetry restoration in the large N_f QCD ($< \frac{11}{2} N_c$) was implied by the fact that the coupling at the infrared fixed point becomes very small [5]. Such a restoration was indeed observed by various methods like lattice simulation [6], ladder Schwinger-Dyson equation [7, 8], dispersion relation [9], instanton calculus [10], etc.

In Ref. 1 we pointed that the chiral restoration takes place for large N_f also in the HLS *by its own dynamics*. Inclusion of the quadratic divergences in the renormalization group equations (RGE's) was essential to obtain the phase transition. Here I should emphasize that *thanks to the gauge symmetry* in the HLS it is possible to perform a *systematic loop expansion* including the vector mesons in addition to the pseudoscalar mesons [11, 12, 13, 14, 2] in a way to extend the chiral perturbation theory [15, 16]. There the loop expansion corresponds to the derivative expansion, so that the one-loop calculation of the RGE is reliable in the low-energy region. The quadratic divergence in the RGE's yields the quadratic running of (square of) the decay constant $F_\pi^2(\mu)$, where μ is the renormalization point. What is shown in Ref. 1 is that *the order parameter $F_\pi(0)$ can become zero for larger N_f even when $F_\pi(\Lambda) \neq 0$*, where $F_\pi(\Lambda)$ is not the order parameter but just a parameter of the bare Lagrangian defined at the cutoff Λ where the matching with QCD is made.

In Ref. 2 we proposed a novel way of matching the HLS with the underlying QCD in the sense of a Wilsonian RGE, namely, including quadratic divergences in the HLS ("Wilsonian matching"). The basic tool of the Wilsonian matching is the Operator Product Expansion (OPE) of QCD for the axialvector and vector current correlators, which are equated with those from the HLS at the matching scale Λ. *This determines without much ambiguity the bare parameters of the HLS defined at the scale Λ in terms of the QCD parameters.* It is shown that the physical quantities for the π and ρ system are calculated by the Wilsonian RGE's from the bare parameters in remarkable agreement with experiment.

In Ref. 3 we applied the Wilsonian matching for the large N_f QCD, and proposed "Vector Manifestation" (VM) of the chiral symmetry as a novel manifestation of the Wigner realization in which *the vector meson denoted by ρ (ρ meson and its flavor partner) becomes massless at the chiral phase transition point*. Accordingly, *the (longitudinal) ρ becomes the chiral partner of the Nambu-Goldstone (NG) boson denoted by π (pion and its flavor partners)*.

Below I shall summarize main points of the Wilsonian matching and the VM.

Let me start from the Wilsonian matching [2] of the HLS with the underlying QCD. In the HLS axialvector and vector current correlators are well described by the tree contributions with including $\mathcal{O}(p^4)$ terms when the momentum is around the matching scale, $Q^2 \sim \Lambda^2$:

$$\Pi_A^{(\text{HLS})}(Q^2) = \frac{F_\pi^2(\Lambda)}{Q^2} - 2z_2(\Lambda) \ , \quad \Pi_V^{(\text{HLS})}(Q^2) = \frac{F_\sigma^2(\Lambda)\left[1 - 2g^2(\Lambda)z_3(\Lambda)\right]}{M_v^2(\Lambda) + Q^2} - 2z_1(\Lambda) \ , \tag{1}$$

where $g(\Lambda)$ is the bare HLS gauge coupling, $F_\sigma^2(\Lambda) = a(\Lambda)F_\pi^2(\Lambda)$ the bare decay constant of the would-be NG boson σ absorbed into the HLS gauge boson ρ, and $M_v^2(\Lambda) \equiv g^2(\Lambda)F_\sigma^2(\Lambda)$ the bare ρ mass. $z_1(\Lambda)$, $z_2(\Lambda)$ and $z_3(\Lambda)$ are the (bare) coefficients of the relevant $\mathcal{O}(p^4)$ terms. The same correlators are evaluated by the OPE up until $\mathcal{O}(1/Q^6)$ [17]:

$$\Pi_A^{(\text{QCD})}(Q^2) = \frac{1}{8\pi^2}\left[-\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{Q^2}{\mu^2} + \frac{\pi^2}{3}\frac{\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle}{Q^4} + \frac{\pi^3}{3}\frac{1408}{27}\frac{\alpha_s\langle\bar{q}q\rangle^2}{Q^6} \right] \ ,$$

$$\Pi_V^{(\text{QCD})}(Q^2) = \frac{1}{8\pi^2}\left[-\left(1 + \frac{\alpha_s}{\pi}\right)\ln\frac{Q^2}{\mu^2} + \frac{\pi^2}{3}\frac{\langle\frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu}\rangle}{Q^4} - \frac{\pi^3}{3}\frac{896}{27}\frac{\alpha_s\langle\bar{q}q\rangle^2}{Q^6} \right] \ , \tag{2}$$

where μ is the renormalization scale of QCD. The current correlators in the HLS in Eq. (1) can be matched with those in QCD in Eq. (2) up until first derivatives at Λ:

$$\Pi_A^{(\text{HLS})} - \Pi_V^{(\text{HLS})} = \Pi_A^{(\text{QCD})} - \Pi_V^{(\text{QCD})} \ , \quad \frac{d}{dQ^2}\Pi_{A,V}^{(\text{HLS})} = \frac{d}{dQ^2}\Pi_{A,V}^{(\text{QCD})} \ , \quad \text{at } Q^2 = \Lambda^2 \ . \tag{3}$$

Here the difference of two current correlators in the first equation is taken to eliminate the explicit dependence on the renormalization scale μ of QCD. The above three equations are the Wilsonian matching conditions proposed in Ref. 2.

The right-hand sides in three equations in (3) are directly determined from QCD. First note that the matching scale Λ must be small enough for the validity of the systematic expansion in the HLS, whereas Λ has to be big enough for the OPE to be valid. Here I take $\Lambda = 1.1\,\text{GeV}$. To determine the current correlators from the OPE we used

$$\left\langle \frac{\alpha_s}{\pi}G_{\mu\nu}G^{\mu\nu} \right\rangle = 0.012\,\text{GeV}^4 \ , \quad \langle\bar{q}q\rangle_{1\,\text{GeV}} = -\left(0.25\,\text{GeV}\right)^3 \ , \tag{4}$$

shown in Ref. 17 and $\Lambda_{\text{QCD}} = 400\,\text{MeV}$ as typical values. One-loop running is used to estimate $\alpha_s(\Lambda)$ and $\langle\bar{q}q\rangle_\Lambda$. Then the bare parameters $F_\pi(\Lambda)$, $a(\Lambda)$, $g(\Lambda)$, $z_3(\Lambda)$ and $z_2(\Lambda) - z_1(\Lambda)$ can be determined through the Wilsonian matching conditions. Actually, the Wilsonian matching conditions in Eq. (3) are not enough to determine all the relevant bare parameters. Therefore, the on-shell pion decay constant $F_\pi(0) = 88\,\text{MeV}$ in the chiral limit [16] and the ρ mass $m_\rho = 770\,\text{MeV}$ are used as inputs. The resultant values of all the relevant bare parameters of the HLS are shown in Table 1 together with those at $\mu = m_\rho$.

μ	$F_\pi(\mu)$	$a(\mu)$	$g(\mu)$	$z_3(\mu)$	$z_2(\mu) - z_1(\mu)$
$\Lambda = 1.1\,\text{GeV}$	0.149	1.19	3.69	-5.23×10^{-3}	-1.03×10^{-3}
$m_\rho = 0.77\,\text{GeV}$	0.110	1.22	6.33	-6.34×10^{-3}	-1.24×10^{-3}

Table 1: Five parameters of the HLS at $\mu = \Lambda$ and m_ρ for $\Lambda_{\text{QCD}} = 400\,\text{MeV}$. The unit of F_π is GeV.

Now that the bare Lagrangian has been completely specified, several physical quantities can be predicted by the Wilsonian RGE's. Table 2 shows two examples of the physical predictions given in

	$\Gamma(\rho \to \pi\pi)$	$\Gamma(\rho \to e^+e^-)$
theory	151 MeV	6.8,keV
Exp.	(150.8 ± 2.0) MeV	(6.77 ± 0.32) keV

Table 2: Predictions of $\Gamma(\rho \to \pi\pi)$ and $\Gamma(\rho \to e^+e^-)$ from the Wilsonian matching.

Ref. 2. The agreement between the predicted and experimental values are excellent. Especially, the prediction of $\Gamma(\rho \to e^+e^-)$ is substantially improved from that at the leading order, $\Gamma(\rho \to e^+e^-) = 5.2$ keV. (See, e.g. Ref. 18, where a tree-level analysis of the equivalent model [19] was made.) This shows that the Wilsonian matching works very well for the $N_f = 3$ QCD.

Now, let me briefly summarize the VM proposed in Ref. 3. One important result of the Wilsonian matching is that $F_\pi^2(\Lambda)$ *is non-zero even at the critical point* where the chiral symmetry is restored with $\langle \bar{q}q \rangle = 0$. Then how do we know by the bare parameters defined at Λ whether or not the chiral symmetry is restored ? A clue comes from the fact that $\Pi_A^{\rm (QCD)}$ and $\Pi_V^{\rm (QCD)}$ in Eq. (2) agree with each other for any value of Q^2 at the critical point. Thus, through the Wilsonian matching, it is reasonable to require that $\Pi_A^{\rm (HLS)}$ and $\Pi_V^{\rm (HLS)}$ in Eq. (1) agree with each other for *any value of Q^2*. This agreement is satisfied by the following conditions:

$$g(\Lambda) \to 0 \ , \quad a(\Lambda) = F_\sigma^2(\Lambda)/F_\pi^2(\Lambda) \to 1 \ , \quad z_1(\Lambda) - z_2(\Lambda) \to 0 \ . \tag{5}$$

The low-energy phenomena are studied by solving the RGE's. We found that the on-shell pion decay constant $F_\pi(0)$, which is actually the order parameter, vanishes even though it at the matching scale $F_\pi(\Lambda)$ is non-zero. The critical value of N_f is expressed by the parameters in the OPE as

$$N_f^{\rm cr} = 4 \left[1 + \frac{\alpha_s}{\pi} + \frac{2\pi^2}{3} \frac{\langle \frac{\alpha_s}{\pi} G_{\mu\nu} G^{\mu\nu} \rangle}{\Lambda^4} \right] \ . \tag{6}$$

For wide range of the values of the parameters this is estimated as $N_f^{\rm cr} \simeq 5$. Furthermore, since $(g, a) = (0, 1)$ is the fixed point of the RGE's [1], $g(m_\rho) = 0$ and $a(m_\rho) = 1$ at the ρ mass scale m_ρ. This implies that ρ becomes massless ($m_\rho = g(m_\rho)F_\sigma(m_\rho) = 0$) with the current coupling equal to that of π ($F_\sigma(m_\rho) = F_\pi(0)$):

$$m_\rho \to 0 \ , \quad F_\sigma(m_\rho) \to F_\pi(0) \quad \text{for} \quad N_f \to N_f^{\rm cr} \ . \tag{7}$$

This is nothing but the VM of the chiral symmetry which is accompanied by the degenerate massless π and (longitudinal) ρ. A salient feature of the VM is that m_ρ *approaches to zero faster than* F_π; [3]

$$m_\rho^2/F_\pi^2(0) \to a(m_\rho)g^2(m_\rho) \to 0 \ . \tag{8}$$

Finally, let me make a comment on the application of the VM to other chiral phase transitions such as the one at finite temperature and/or density [20]. In such a case, the position of the ρ peak of the dilepton spectrum would move to the lower energy region in accord with the picture shown in Ref. 20. Furthermore, the VM would imply smaller ρ width ($\Gamma/m_\rho \sim g_{\rho\pi\pi}^2 \to 0$) near the critical point. If it is really the case, this would be clear signals of VM tested in the future experiments.

Acknowledgment

References

[1] M. Harada and K. Yamawaki, Phys. Rev. Lett. **83**, 3374 (1999).

[2] M. Harada and K. Yamawaki, hep-ph/0009163 (To be published in Phys. Rev. D.).

[3] M. Harada and K. Yamawaki, Phys. Rev. Lett. **86**, 757 (2001).

[4] M. Bando, T. Kugo, S. Uehara, K. Yamawaki and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando, T. Kugo and K. Yamawaki, Phys. Rept. **164**, 217 (1988).

[5] T. Banks and A. Zaks, Nucl. Phys. B **196**, 189 (1982).

[6] J. B. Kogut and D. K. Sinclair, Nucl. Phys. B **295**, 465 (1988); F. R. Brown, H. Chen, N. H. Christ, Z. Dong, R. D. Mawhinney, W. Schaffer and A. Vaccarino, Phys. Rev. D **46**, 5655 (1992); Y. Iwasaki, K. Kanaya, S. Sakai and T. Yoshie, Phys. Rev. Lett. **69**, 21 (1992); Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai and T. Yoshie, Nucl. Phys. Proc. Suppl. **53**, 449 (1997); Prog. Theor. Phys. Suppl. **131**, 415 (1998).

[7] T. Appelquist, J. Terning and L.C. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996); T. Appelquist, A. Ratnaweera, J. Terning and L. C. Wijewardhana, Phys. Rev. D **58**, 105017 (1998).

[8] V.A. Miransky and K. Yamawaki, Phys. Rev. D **55**, 5051 (1997) [Erratum-ibid. D **56**, 3768 (1997)].

[9] R. Oehme and W. Zimmermann, Phys. Rev. D **21**, 471 (1980).

[10] M. Velkovsky and E. Shuryak, Phys. Lett. B **437**, 398 (1998).

[11] H. Georgi, Phys. Rev. Lett. **63**, 1917 (1989); Nucl. Phys. B **331**, 311 (1990).

[12] M. Tanabashi, Phys. Lett. B **316**, 534 (1993).

[13] M. Harada and K. Yamawaki, Phys. Lett. B **297**, 151 (1992).

[14] M. Harada, T. Kugo and K. Yamawaki, Phys. Rev. Lett. **71**, 1299 (1993); Prog. Theor. Phys. **91**, 801 (1994).

[15] S. Weinberg, Physica A **96**, 327 (1979).

[16] J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984); Nucl. Phys. B **250**, 465 (1985).

[17] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **147**, 385 (1979); Nucl. Phys. B **147**, 448 (1979).

[18] M. Harada and J. Schechter, Phys. Rev. D **54**, 3394 (1996).

[19] O. Kaymakcalan and J. Schechter, Phys. Rev. D **31**, 1109 (1985).

[20] G.E. Brown and M. Rho, Phys. Rev. Lett. **66**, 2720 (1991); Phys. Rept. **269**, 333 (1996); nucl-th/0101015; hep-ph/0103102.